Level – 2	Daily Tutorial Sheet - 9

111.(A) Put $x = z^3 \Rightarrow dx = 3z^2 dz \quad \therefore \quad \int \frac{dx}{x\left(1+\sqrt[3]{x}\right)^2} = \int \frac{3z^2 dz}{z^3(1+z)^2} = 3\int \frac{dz}{z(z+1)^2}$

$= 3\int \left[\frac{1}{z} - \frac{1}{1+z} - \frac{1}{(1+z)^2}\right] dz$ [by partial fraction]

$= 3\left[\log z - \log(1+z) + \frac{1}{(1+z)}\right] + C = 3\left(\log \frac{x^{1/3}}{1+x^{1/3}} + \frac{1}{1+x^{1/3}}\right) + C$

112.(B) Put $1 + x^2 = x^2 z^4 \Rightarrow x^2 = \frac{1}{z^4 - 1} \Rightarrow dx = \frac{-2z^3}{x(z^4-1)^2} dz \quad \therefore \quad \int \frac{dx}{x^{1/2}(1+x^2)^{5/4}}$

$= \int \frac{1}{x^{1/2}(x^2 z^4)^{5/4}} \left(\frac{-2z^3}{x(z^4-1)^2}\right) dz = -2\int \frac{1}{x^{1/2}(x^2 z^4)^{5/4}} \left(\frac{-2z^3}{x(z^4-1)^2}\right) dz$

$= -2\int \frac{dz}{x^4 (z^4-1)^2 z^2} = -2\int \frac{dz}{z^2} = \frac{2}{z} + C = \frac{2\sqrt{x}}{\sqrt[4]{1+x^2}} + C.$

113.(B) Put $1 + x = \frac{1}{z} \Rightarrow dx = -\frac{1}{z^2} dz \quad \therefore \quad \int \frac{dx}{(1+x)^5 \sqrt{x^2+2x}} = -\int \frac{1}{z^2} \cdot \frac{z^5 dz}{\sqrt{\frac{(1-z)^2}{z^2} + \frac{2(1-z)}{z}}}$

$= -\int \frac{z^4}{\sqrt{1-z^2}} dz = -\int \sin^4 \theta \, d\theta$ [Putting $z = \sin\theta \Rightarrow dz = \cos\theta \, d\theta$]

114.(B) $I = \int \frac{\left(\frac{2}{x^2} + \frac{1}{x\sqrt{x}}\right) dx}{\left(1 + \frac{1}{\sqrt{x}} + \frac{1}{x}\right)^2}$

Let $1 + \frac{1}{\sqrt{x}} + \frac{1}{x} = t \Rightarrow I = -2\int \frac{dt}{t^2} = \frac{2}{t} + c = \frac{2x}{x + \sqrt{x} + 1} + c = f(x)$

$f(0) = 0 \Rightarrow c = 0 \Rightarrow [f(4)] = \left[\frac{8}{7}\right] = 1$

115.(C) Let $I = \int \frac{(5x^4 + 4x^5)}{(x^5 + x + 1)^2} dx$

Dividing above and below by x^{10}, we get: $I = \frac{\left(\frac{5}{x^6} + \frac{4}{x^5}\right) dx}{\left(1 + \frac{1}{x^4} + \frac{1}{x^5}\right)^2}$

Putting $1 + \frac{1}{x^4} + \frac{1}{x^5} = t$, $\left(-\frac{4}{x^5} - \frac{5}{x^6}\right) dx = dt$ or $\left(\frac{4}{x^5} + \frac{5}{x^6}\right) dx = -dt$, we get:

$$I = -\int \frac{dt}{t^2} = \frac{1}{t} + c = \frac{1}{\left(1 + \frac{1}{x^4} + \frac{1}{x^5}\right)} + c \qquad = \frac{x^5}{(x^5 + x + 1)} + c = f(x) \quad f(0) = 0 \Rightarrow c = 0 \Rightarrow f(1) = \frac{1}{3}$$

116.(D) $I = \int \left(x^{3t} + x^{2t} + x^t\right)\left(2x^{2t} + 3x^t + 6\right)^{1/t} dx = \int \left(x^{3t-1} + x^{2t-1} + x^{t-1}\right) x \left(2x^{2t} + 3x^t + 6\right)^{1/t} dx$

$= \int \left(x^{3t-1} + x^{2t-1} + x^{t-1}\right)\left(2x^{3t} + 3x^{2t} + 6x^t\right)^{1/t} dx$

Put $\left(2x^{3t} + 3x^{2t} + 6x^t\right) = u$ and $6t\left(x^{3t-1} + x^{2t-1} + x^{t-1}\right)dx = du$

$\Rightarrow \qquad I = \int \frac{(u)^{1/t} du}{6t} = \frac{(u)^{1/t+1}}{6t\left(\frac{1}{t} + 1\right)} = \frac{\left(2x^{3t} + 3x^{2t} + 6x^t\right)}{6(1+t)}$

117.(A) Putting, $I^{r+1}(x) = t$ and $\frac{1}{xI(x)\,I^2(x)\ldots I^r(x)}dx = dt$, we get:

$$\int \frac{1dx}{xI^2(x)I^3(x)\ldots I^r(x)} = \int 1\,.\,dt = t + C = I^{r+1}(x) + C$$

118.(C) $\int e^{\sec x}\left(\tan^2 x + \sec x + \sec^2 x + \tan x \sec^2 x + \tan x \sec x\right)dx$

$= \int e^{\sec x}\left[\sec x \tan x\left(\sec x + \tan x\right) + \left(\sec x \tan x + \sec^2 x\right)\right]dx$

$= \int e^{\sec x} \sec x \tan x\,.\left(\sec x + \tan x\right)dx + \int e^{\sec x}\,.\left(\sec x \tan x + \sec^2 x\right)dx$

$= \left(\sec x + \tan x\right)e^{\sec x} - \int \left(\sec x \tan x + \sec^2 x\right)e^{\sec x}dx + \int e^{\sec x}\left(\sec x \tan x + \sec^2 x\right)dx$

$= e^{\sec x}\left(\sec x + \tan x\right) + C$

119.(A) $\int \frac{ax^2 - b}{x\sqrt{c^2x^2 - \left(ax^2 + b\right)^2}}dx = \int \frac{a - b/x^2}{\sqrt{c^2 - \left(ax + \frac{b}{x}\right)^2}}dx = \int \frac{1}{\sqrt{c^2 - \left(ax + \frac{b}{x}\right)^2}}\,d\left(ax + \frac{b}{x}\right) = \sin^{-1}\left(\frac{ax + \frac{b}{x}}{c}\right) + k$

120.(D) $I = \int \frac{\left(x^2 - 1\right)dx}{x^3\sqrt{2x^4 - 2x^2 + 1}} = \int \frac{\left(\frac{1}{x^3} - \frac{1}{x^5}\right)dx}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}}$ Put $2 - \frac{2}{x^2} + \frac{1}{x^4} = t^2$ to get:

$$I = \int \frac{\frac{1}{4}2t\,dt}{t} = \frac{t}{2} + C = \frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$$